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IS SPONTANEOUS SWIRLING OF AXISYMMETRIC FLOW POSSIBLE?

B. A. Lugovtsov

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References [1-4] proposed a hypothesis on the possibility of the appearance of spontaneous swirling of an axisymmetric flow in the absence of explicit sources of rotation, when axisymmetric flow without rotation is clearly possible. The replacement of the symmetry type (axial symmetry/rotational-axial symmetry) is associated with bifurcation of the starting regime at some Reynolds number, when the equation for the rotational component admits nontrivial solutions, corresponding to the stable regime, and the regime without rotation becomes unstable. In the steady-state flow an initial disturbance is completely forgotten, and the intensity of the rotation is independent of the initial disturbance (however the direction of rotation is determined by the initial disturbance). The authors of [4] term this phenomenon an autorotation or a vortical dynamo.

The proposed hypothesis is extremely interesting, may have a significant influence on the approach to the explanation of many natural phenomena, and has a fundamental nature. Therefore it is very important to obtain convincing proof of the validity or incorrectness of this proposal.

We shall present arguments that yield a basis for serious doubts concerning the validity of the proposed hypothesis and the existence of the spontaneous swirling phenomenon.

If the fluid viscosity is constant (independent of the coordinates) and the fluid motion is laminar, stationary, and rotationally symmetric, then the equation for $\Gamma = rv_\varphi$ in a cylindrical coordinate system with the axis of symmetry z has the form (in the conventional notations)

$$v_r \frac{\partial \Gamma}{\partial r} + v_z \frac{\partial \Gamma}{\partial z} = \nu \left(\frac{\partial^2 \Gamma}{\partial r^2} - \frac{1}{r}\frac{\partial \Gamma}{\partial r} + \frac{\partial^2 \Gamma}{\partial z^2} \right). \quad (1)$$

The two-sided maximum principle is valid for this equation: the maximum and minimum of Γ are reached on the boundary. This circumstance was noted in [3, 4]. This implies that spontaneous swirling is not possible for stationary axisymmetric laminar flows with the condition $\Gamma = 0$ on the boundary, which is a surface of revolution.

However, it is stated in [4] that if the conditions of the absence of rotational flow are specified on part of the boundary, then spontaneous swirling is possible even in a homogeneous fluid. An example of such flow is presented in [3], where the problem is examined of the flow between a porous rotating disc and a plane surface.

We can show that this statement is erroneous. Spontaneous swirling does not arise even if on part of the boundary condition $\Gamma = 0$ is specified, and on the boundary segment which is a free surface there are given the conditions

$$\tau_\varphi = \sigma_{\varphi r} n_r + \sigma_{\varphi z} n_z = 0, \; v_r n_r + v_z n_z = 0, \quad (2)$$

where τ_φ is the azimuthal component of the tangential stress vector on the free surface; $n = (n_r, 0, n_z)$ is the outward normal to this surface;

$$\sigma_{\varphi r} = \frac{\eta}{r}\left(\frac{\partial \Gamma}{\partial r} - \frac{2\Gamma}{r} \right), \; \sigma_{\varphi z} = \frac{\eta}{r}\frac{\partial \Gamma}{\partial z}$$

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are the components of the viscous stress tensor; η is the coefficient of dynamic viscosity.

In fact, multiplying Eq. (1) by r and integrating it in the meridional plane over the region bounded by the symmetry axis and the boundary 1, we obtain

$$\oint_l \left(r\frac{\partial\Gamma}{\partial n} - 2\Gamma n_r\right) dl = 0.$$

On the boundary segments, where $\tau_\varphi = 0$, and on the symmetry axis the integrand vanishes identically, and on the boundary segments l' (the segment length l' is nonzero), where $\Gamma = 0$, we have

$$\int_{l'} r\frac{\partial\Gamma}{\partial n} dl = 0.$$

This implies that within the region there will be a line on which $\Gamma = 0$ and which divides the initial region into two regions, each of which is equivalent to the initial region from the viewpoint of the boundary conditions.

Repetition of the described procedure for these regions leads to division of each of them by the line $\Gamma = 0$ into two regions. Performing this procedure repeatedly, we find that $\Gamma \equiv 0$ within the flow region.

We note that the boundary condition (2) and the condition $\Gamma = 0$ on l' provide the necessary control of the axial component of the fluid angular momentum, for which it makes sense to speak of spontaneous swirling. If we drop the free-surface nonpenetration condition ($v_r n_r + v_z n_z \neq 0$) in Eq. (2), then an axial component of the angular momentum can be introduced externally, and in this case it makes no sense to speak of autorotation, It is this case that is examined in [3].

Thus, the appearance of spontaneous swirling in the case of laminar axisymmetric flow of a homogeneous viscous fluid is not possible.

In [1, 2] it is proposed that with the development of instability and the appearance of stationary turbulent flow a mechanism is possible, as a result of which there arises, at least in the near-axis region, a swirled jet, having an axial component of the angular momentum.

To substantiate this proposal, a study was made in [1, 2] of the stability of a model flow in an infinite fluid, corresponding to a circular jet issuing from a pipe into a fluid that is at rest at infinity. It was found that under certain conditions a swirled jet appears.

We shall consider this example for fully developed turbulent flow. It is natural to presume that the averaged flow is stationary, has rotational symmetry, and in the cylindrical coordinate system (r, φ, z) has the form $v = [0, v_\varphi(r), v_z(r)]$. Parallelism of the averaged flow is ensured by the presence of the corresponding mass forces, which maintain the laminar profile $v_z = U(r)$ of the initial flow. Assuming that the turbulent nature of the flow can be taken into account by introducing the turbulent viscosity $\nu_i = \nu_i(r) > 0$, and introducing the notation $\nu_* = \nu_*(r) = \nu + \nu_t(r)$, we obtain the equation for Γ

$$\frac{\partial}{\partial r} r\nu_* \left(\frac{\partial\Gamma}{\partial r} - \frac{2\Gamma}{r}\right) = 0$$

with the boundary conditions $\Gamma(0) = \Gamma(\infty) = 0$. It is not difficult to see that the only solution of this equation that satisfies the boundary conditions is $\Gamma = 0$, i.e., flow without swirl, which does not agree with the results of [2].

We shall examine axisymmetric flow within a region that is boundary by a porous surface of revolution, on which $\Gamma = 0$, but the fluid can flow through this boundary (v_r and v_z are specified so that the condition ensuring mass conservation is satisfied). Then, using for the description of the turbulent flow the Navier–Stokes equations with the turbulent viscosity coefficient, we obtain for Γ the equation

$$\frac{\partial r v_r \Gamma}{\partial r} + \frac{\partial r v_z \Gamma}{\partial r} = \frac{\partial}{\partial r} r\nu_* \left(\frac{\partial\Gamma}{\partial r} - \frac{2\Gamma}{r}\right) + \frac{\partial}{\partial z} r\nu_* \frac{\partial\Gamma}{\partial z},$$

integrating which in the meridional plane over the region bounded by the symmetry axis and the boundary, we have

$$\oint r\nu_* \frac{\partial\Gamma}{\partial n} dl = 0,$$

from which follows the existence of the line $\Gamma = 0$ within the region. Repeating this process, just as in the constant viscosity case, we come to the conclusion that $\Gamma \equiv 0$ within the flow region being examined, i.e., spontaneous swirling is absent.

This result contradicts the subject hypothesis and yields a basis for questioning its validity.

We note that the results of [3, 4], obtained in the case of self-similar conical flows with turbulent viscosity, can not be considered to be examples of the existence of the spontaneous swirling (or autorotation) phenomenon, since in these examples Γ remains finite at infinity, and redistribution of the swirl takes place rather than the appearance of swirl.

We shall examine as a close analog of the conical flows the Burgers turbulent vortex, assuming that the turbulent viscosity and, consequently, $\nu_* = \nu_*(r)$ depend only on r. In this case the solution is

$$v_r = -ar,\ v_z = 2az,\ \Gamma = \Gamma(r),$$

and for Γ we obtain the equation

$$-ar^2 \frac{\partial}{\partial r} r^2\omega = \frac{\partial}{\partial r} \nu_* r^3 \frac{\partial \omega}{\partial r}, \tag{3}$$

where $\omega = \Gamma/r^2$.

Let at infinity $\Gamma(\infty) = 0$. Then $\omega r^2 \to 0$ as $r \to \infty$ and $\omega(0) = \omega_0$. We multiply Eq. (3) by ω. As a result we have

$$-\frac{\partial}{\partial r}\frac{1}{2} ar^4\omega^2 = \frac{\partial}{\partial r} \nu_* r^3 \frac{\partial \omega}{\partial r} - \nu_* r^3 \left(\frac{\partial \omega}{\partial r}\right)^2. \tag{4}$$

Integrating Eq. (4) with respect to r from 0 to ∞, with account for the boundary conditions we obtain

$$\int_0^\infty \nu_* r^3 \left(\frac{\partial \omega}{\partial r}\right)^2 dr = 0,$$

from which follows $\omega \equiv 0$, i.e., spontaneous swirling is absent in this case also.

Thus, although the question remains open for the more complex models of closure of the turbulent flow equations, the phenomenon of spontaneous swirling of the axisymmetric flow apparently does not take place.

Experimental confirmation or contradiction of the proposed hypothesis is very problematic. The primary difficulty is associated with the fact that swirling may arise as a result of deviation from exact symmetry in real systems, which is quite difficult to control. The experimental data available at the present time do not argue in favor of spontaneous swirling. For example, it is well known that a vortex does not arise in a bathtub with careful observance of the necessary symmetry conditions.

We note that if on the boundary of the region or as $r \to \infty$ for Eq. (3) Γ has a nonzero value, then, since the maximum principle in the variable viscosity case, generally speaking, does not hold, within the region Γ may take values that are larger than on the boundary. This could be treated just as spontaneous swirling.

In this connection we can formulate as an alternative to the subject hypothesis the assumption that the dependence of the turbulent viscosity on the coordinates must by such that the maximum principle holds.

In conclusion we note that in principle the bifurcation of an initial axisymmetric flow into nonaxisymmetric flow with rotation is possible, where $\Gamma = \Gamma(r, \varphi, z) \not\equiv 0$ with $\Gamma = 0$ on the axisymmetric boundary. In this case the analog of the absence of spontaneous swirling would be satisfaction of the condition.

$$\int_0^{2\pi} \Gamma(r, \varphi, z) d\varphi = 0.$$

In the turbulent flow case the requirement for satisfaction of this condition would impose additional conditions on the closure model.

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